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An Application of Assignment Problem in Traveling Salesman Problem (TSP)

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Abstract
Assignment problem (AP) is completely degenerate form of a transportation problem. It appears in some decision-making situations, this paper focused on TSP for finding the shortest closed route. By using 'ROA Method' and 'Ghadle and Muley Rule' will get optimal solution for TSP within few steps.
Keyword- Assignment problem, TSP, linear integer programming, Revised Ones Assignment Method (ROA).

I. Introduction

The Traveling Salesman Problem (TSP) is a classical combinatorial optimization problem, which is simple to state but very difficult to solve. The problem is to find the shortest possible tour through a set of N vertices so that each vertex is visited exactly once.

In this paper traveling salesman problem solved like assignment problem using linear programming approach. The constraints require that the salesman must enter and leave each city exactly once.

II. Mathematical Formulation of Traveling Salesman Problem (TSP)

Let 1, 2 … n be the labels of the n cities and $C = C_{i,j}$ be an n x n cost matrix where $C_{i,j}$ denotes the cost of traveling from city i to city j. Then, the general formulation of the traveling salesman problem (TSP), as described by Assignment Problem, is shown below.

	1	2	3	…	n
1	∞	C_{12}	C_{13}	…	C_{1n}
2	C_{21}	∞	C_{23}	…	C_{2n}
3	C_{31}	C_{32}	∞	…	C_{3n}
⋮	⋮	⋮	⋮	∞	⋮
n	C_{n1}	C_{n2}	C_{n3}	…	∞

If $C_{i,j} = C_{j,i}$, the problem is called symmetric traveling salesman problem (STSP).

III. ASSIGNMENT BASED FORMULATION

Starting from his home, a salesman wishes to visit each of (n−1) other cities and return home at minimal cost. He must visit each city exactly once and it costs C_{ij} to travel from city i to city j.
We may be tempted to formulate his problem as the assignment problem:

$$X_{ij} = \begin{cases} 1 ; & \text{if he goes from city i to city j.} \\ 0 ; & \text{otherwise.} \end{cases}$$

Then the mathematical formulation of the assignment problem is,
Subject to the constraints,

$$\text{Minimize } \quad Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} X_{ij} \qquad \text{.......(1)}$$

$$\sum_{i=1}^{n} X_{ij} = 1 \text{ and } \sum_{j=1}^{n} X_{ij} = 1 \ : \ X_{ij} = 0 \text{ or } 1 \qquad \text{.......(2)}$$

for all i=1,2,....n and j=1,2,.....n.

IV. Revised Ones Assignment Method (ROA) for Solving Assignment Problem. [1, 2]

This section presents a method to solve the assignment problem which is different from the preceding method. We call it "Revised Ones Assignment Method" because of making assignment in terms of ones.

This method is based on creating some ones in the assignment matrix and then tries to find a complete assignment in terms of ones. By a complete assignment we mean an assignment plan containing exactly n assigned independent ones, one in each row and one in each column.

Now, consider the assignment matrix where C_{ij} is the cost or effectiveness of assigning i^{th} machine.

$$\begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ n \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & n \end{array} \\ \begin{pmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & \cdots & C_{nn} \end{pmatrix} \end{array}$$

The new algorithm is as follows.
Let (1-2) be an assignment problem in which the objective function can be minimized or maximized.

Step 1

In a minimization (maximization) case, find the minimum (maximum) element of each row in the assignment matrix (say a_i) and write it on the right hand side of the matrix.

$$\begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & n \end{array} \\ \begin{pmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & \cdots & C_{nn} \end{pmatrix} \end{array}\begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{array}$$

Then divide each element of i^{th} row of the matrix by a_i. These operations create at least one ones in each rows.
In term of ones for each row and column, do assignment. Otherwise go to step 2.

Step 2

Find the minimum (maximum) element of each column in assignment matrix (say b_j), and write it below j^{th} column. Then divide each element of j^{th} column of the matrix by b_j.
These operations create at least one ones in each columns. Make assignment in terms of ones. If no feasible assignment can be achieved from step (1) and (2) then go to step 3.

$$\begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ n \\ \\ \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & n \end{array} \\ \begin{pmatrix} C_{11}/a_1 & C_{12}/a_1 & \cdots & C_{1n}/a_1 \\ C_{21}/a_2 & C_{22}/a_2 & \cdots & C_{2n}/a_2 \\ \vdots & \vdots & \ddots & \vdots \\ C_{n1}/a_n & C_{n2}/a_n & \cdots & C_{nn}/a_n \end{pmatrix} \\ \begin{array}{cccc} b_1 & b_2 & \cdots & b_n \end{array} \end{array}$$

Step 3

Draw the minimum number of lines to cover all the ones of the matrix. If the number of drawn lines less than n, then the complete assignment is not possible, while if the number of lines is exactly equal to n, then the complete assignment is obtained.

Step 4

If a complete assignment program is not possible in step 3, then select the smallest (largest) element (say d_{ij}) out of those which do not lie on any of the lines in the above matrix. Then divide by d_{ij} each element of the uncovered rows or columns, which d_{ij} lies on it. This operation creates some new ones to this row or column.
If still a complete optimal assignment is not achieved in this new matrix, then use step 4 and 3 iteratively. By repeating the same procedure the optimal assignment will be obtained.
(To assign one we have add Step 5 which is mentioned below.)

Step 5 (Ghadle and Muley Rule) [3]

i) For minimization problem select max number from calculated matrix and write it on right hand side as well as bottom side.
  - To assign one, start from min number of columns (bottom side) and select ones.
  - If there are more than one ones in any column then ignore temporarily, and give last priority to that column.
  - If still there are identical ones in column then give the priority to max number of rows (right hand side).
  - If there are more than one ones in any row then give first come priority.

Or

ii) For maximization problem select min number from calculated matrix and write it on right hand side as well as bottom side.
  - To assign one, start from max number of columns (bottom side) and select ones.
  - If there are more than one ones in any column then ignore temporarily, and give last priority to that column.
  - If still there are identical ones in column then give the priority to min number of rows (right hand side).
- If there are more than one ones in any row then give first come priority.

Priority rule

One question arises here. What to do with non square matrix? To make square, a non square matrix, we add one artificial row or column which all elements are one. Thus we solve the problem with the new matrix, by using the new method. The matrix after performing the steps reduces to a matrix which has ones in each rows and columns. So, the optimal assignment has been reached.

V. Applications

The Ashtavinayaka Yatra is the most important worship yatra in Maharashtra. Ashtavinayaka means "Eight Ganeshas" when translated in Sanskrit. Lord Ganesha is the Hindu God of unity, remover of obstacles, learning and prosperity.

This yatra covers the eight holy temples of Lord Ganesha which are located around Pune . Each temple has its own individuality and mythology. The deities in each temple are distinct from each other. Some of these deities are described as 'Swayambhu' figurines. This is to point to that no human created these deities; they were formed completely by nature.

According to Hindu Shastra, you will have to visit the Moreshwar temple at Morgaon first and then in sequence visit the temples at Siddhatek, Pali, Mahad, Thevur, Lenyandri, Ozar, and Ranjangaon and in last visit the Moreshwar temple again. This will complete your Ashtavinayak-Yatra.

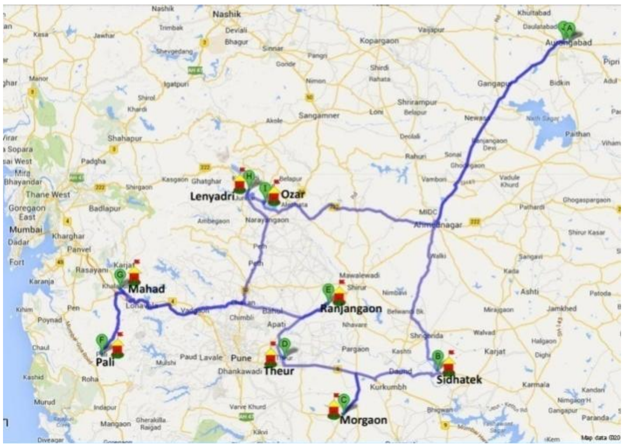
City	Aurangabad	Lenyadri	Mahad	Morgaon	Ozar	Pali	Ranjangaon	Sidhatek	Theur
Aurangabad	-	225	304	236	213	339	187	197	226
Lenyadri	225	-	140	153	15	175	84	160	110
Mahad	304	140	-	152	132	41	121	190	108
Morgaon	236	153	152	-	143	188	70	73	63
Ozar	213	15	132	143	-	166	74	145	102
Pali	339	175	41	188	166	-	157	226	144
Ranjangaon	187	84	121	70	74	157	-	81	43
Sidhatek	197	160	190	73	145	226	81	-	90
Theur	226	110	108	63	102	144	43	90	-

By using 'Revised Ones Assignment Method (ROA)' and 'Ghadle and Muley rule' the final solution obtained was,

		1	2	3	4	5	6	7	8	9
	City	Aurangabad	Lenyadri	Mahad	Morgaon	Ozar	Pali	Ranjangaon	Sidhatek	Theur
1	Aurangabad	-	225	304	236	213	339	187	197	226
2	Lenyadri	225	-	140	153	15	175	84	160	110
3	Mahad	304	140	-	152	132	41	121	190	108
4	Morgaon	236	153	152	-	143	188	70	73	63
5	Ozar	213	15	132	143	-	166	74	145	102
6	Pali	339	175	41	188	166	-	157	226	144
7	Ranjangaon	187	84	121	70	74	157	-	81	43
8	Sidhatek	197	160	190	73	145	226	81	-	90
9	Theur	226	110	108	63	102	144	43	90	-

The solution consist in three cycle which are (1,7),(7,9),(9,4),(4,8),(8,1) ; (2,5),(5,2); (3,6),(6,3) and minimum distance is 675 km.

If we again find next minimum values from column 7th, 5th and 6th , which will cover whole journey of Ashtavinayak Yatra in 987 km.



VI. Conclusions

In this paper, a Revised Ones Assignment Method (ROA) used to assign ones directly and for solving assignment problem as well as for Traveling Salesman Problem. This method can be used for maximize as well as minimized objective functions of Assignment problem.

References

[1] Hadi Basirzadeh, (2012). Ones Assignment Method for solving Assignment Problems, *Applied Mathematical Sciences, Vol. 6, 2012, No. 47*, 2345-2355.

[2] Shweta singh, G.C.Dubey, Rajesh Shrivastava, (August 2012). A Comparative Analysis of Assignment Problem, *IOSR Journal of Engineering (IOSRJEN)*, ISSN: 2250-3021, *Volume 2, Issue 8*, 01-15.

[3] Ghadle Kirtiwant P, Muley Yogesh M (2013), Revised Ones Assignment Method for Solving Assignment Problem, *Journal of Statistics and Mathematics*, ISSN: 0976-8807 & E-ISSN: 0976-8815, *Volume 4, Issue 1*, 147-150.